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D5.1 Development of a Tire – Road Friction Model

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Final version
Project: Integrated Tire and Road Interaction – ITARI
Deliverable 5.1: Development of a Tire-Road Friction Model

Summary
Main objective of work package 5 is to develop a tool for the prediction of the wet grip potential of a road pavement. The following report describes the fundamentals of rubber friction and the developed rubber friction model. Since only wet grip is considered the adhesion part of rubber friction is neglected.

The influence of frequency, temperature and strain on the complex modulus of elasticity of rubber is shown. Since rubber cannot be described by just one single relaxation time it was necessary to extend the formula for the elastic modulus to a summation of terms representing a larger number of relaxation times. This was accomplished by using the Prony series approach. This analytical approach was transferred to a 2D numerical multi-body system consisting of interconnected masses, dampers and springs. The visco-elastic properties of rubber are represented by extended Zener elements (Poynting-Thompson elements). Input quantity is the measured texture profile of the road and output is its friction coefficient.

The parameter identification of the rubber model was performed by using a method suggested by Emri and Tschoegl. Spring and damper constants are strain and temperature dependent. The temperature dependency is allowed for by the WLF equation and the strain dependency is described by the Kraus model. The numerical equations are solved by a time step integration scheme. There are 2 modules allowing for the calculation of the vertical movement of the tire tread into the pavement on the one hand and the horizontal movement (sliding motion over the pavement) on the on the other hand. The first one calculates how deep the rubber tread penetrates into the pavement texture and the second calculates the amount of friction generated.

First results concerning the penetration depth are promising although the model is not validated yet. The horizontal movement is causing difficulties for the differential equation solver at the very moment. We are working on a solution to overcome that problem.
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1. **Introduction**

The main objective of work package 5 is to develop a tool for prediction of the friction potential of a pavement as a function of its texture. Another aim is to establish a definition for the friction potential of a pavement, i.e. the amount of friction that a given pavement is able to provide under definite conditions. Central element to achieve these goals is the development of a tire-road friction model which is able to predict the friction coefficient of a pavement based on its measured texture. The following report describes the fundamentals of rubber friction and the design of the friction model.

2. **Behaviour of Rubber**

When a sinusoidal loading is applied to a piece of rubber like shown in figure 2.1 a phase shift between strain and stress can be observed (a). In the strain-stress diagram a so called hysteresis curve can be observed (b). The area enveloped by the hysteresis curve is a measure for the energy dissipated in the rubber during loading.

![Figure 2.1 hysteresis effect due to sinusoidal loading of a piece of rubber](image)

The phase shift, i.e. the form of the hysteresis curve is a function of the loading frequency. You can imagine that at a very low frequency there will be hardly any phase shift between stress and strain, whereas when you increase the frequency you will notice an increasing phase shift and hysteresis respectively. This frequency-dependent stiffness behavior of the rubber can be described by a complex elastic modulus as given in equation 2.1:

\[
\sigma = E(j\omega) \cdot \varepsilon
\]  

(2.1)

with \(\sigma\) := stress  
\(\varepsilon\) := strain  
\(E(j\omega)\) := frequency-dependent modulus of elasticity.
The complex modulus of elasticity consists of a real part called storage modulus, \( E' \), and an imaginary part called loss modulus, \( E'' \). Their frequency-dependent behavior can be described as follows:

\[
\text{Storage modulus} \quad E'(\omega) = E_0 + (E_\infty - E_0) \frac{(\tau \omega)^2}{1 + (\tau \omega)^2} \tag{2.2}
\]

\[
\text{Loss modulus} \quad E''(\omega) = (E_\infty - E_0) \frac{\tau \omega}{1 + (\tau \omega)^2} \tag{2.3}
\]

with \( \omega \) := angular frequency

\( E_0 \) := relaxation modulus, which defines the stiffness for the static case \( (\omega \to 0) \)

\( E_\infty \) := glassy modulus, which defines the stiffness for very high frequencies \( (\omega \to \infty) \)

\( \tau \) := relaxation time, a constant describing the relaxation characteristics of the rubber

Figure 2.2 shows the two parts of the Young’s modulus for a certain rubber at a temperature of 25 °C. Note that the axes are logarithmically scaled. The storage modulus starts at very low frequencies with the so called “relaxation” modulus and ends up with the so called “glass” modulus at very high frequencies, where the slope of the curve tends to zero. For the loss modulus we observe an increase up to a certain frequency followed by a subsequent decrease. The figure expresses the fact that rubber gets stiffer with in increasing frequency.

Another important property of rubber is that it becomes softer with increasing temperature. This behavior is illustrated in figure 2.3. A rise in temperature is equivalent to a shifting of the curves shown in figure 2.2 towards higher frequencies like indicated in figure 2.3. This temperature-frequency equivalency is known as WLF transformation according to William, Landel und Ferry [1] who originated this kind of transformation in order to describe the temperature behaviour of rubber. A temperature increase results in a shift of the curves describing the modulus of elasticity towards higher frequencies, thus resulting in lower rubber stiffness.

Isac Institute for Road and Traffic Engineering, Technical University of Aachen, March 2006
Another important property of rubber that has to be considered when the behaviour of rubber shall be described is that rubber becomes softer with increasing strain, i.e. the more a piece of rubber is elongated the softer it becomes. This behaviour called “Payne” effect [2] is illustrated in figure 2.4. The Payne effect is non-linear, i.e. it increases with frequency. In figure 2.4 only the real part of the elastic modulus is shown. Its frequency-dependent behaviour is shown for strain rates of 0.1 up to 20 %.

Figure 2.3  Elastic modulus of rubber: frequency shift resulting from temperature increase

Figure 2.4  Elastic modulus of rubber: softening effect due to increasing strain
3. Modelling of Rubber

3.1 Incorporating Different Relaxation Times

When it comes to describing the material behavior of real tire rubber compounds a simple relationship between frequency and elasticity like given in equations (2.1) and (2.2) does no longer apply. The difficulty is that the increase of the elastic modulus spreads over a too large frequency band, which means that the modulus can’t be described by just one single relaxation time $\tau$ and relaxation frequency $2\pi/\tau$ respectively (see figure 3.1 for illustration).

Moreover, the frequency-dependent behavior has to be described by a set of modules, each of them belonging to a certain (relaxation) frequency. One ends up with a summation of Young’s modulus with corresponding relaxation times $\tau_i$, which, at least in the time domain, is called a “Prony” series [3]. The formulation is:

for the storage modulus:

$$E'(\omega) = E_0 + \sum_{i=1}^{m} (E_i - E_{i-1}) \frac{(\tau_i \omega)^2}{1+(\tau_i \omega)^2}$$

(3.1)

for the loss modulus:

$$E''(\omega) = \sum_{i=1}^{m} (E_i - E_{i-1}) \frac{\tau_i \omega}{1+(\tau_i \omega)^2}$$

(3.2)

with

- $\omega$ := angular frequency
- $E_0$ := relaxation modulus, which defines the stiffness for the static case ($\omega \rightarrow 0$)
- $E_i - E_{i-1}$ := relaxation modulus, which defines the stiffness at $\omega_i$
- $\tau_i$ := relaxation time according to angular frequency $\omega_i = 2\pi/\tau_i$
- $m$ := number of elements in Prony series

![Figure 3.1 Elastic modulus of rubber: function cannot be described by just one relaxation frequency](image-url)
3.2 Modelling Rubber

The model to be developed is meant to focus on the skid resistance of road pavements. By definition skid resistance is the resistance a pavement can offer to a braking car under wet conditions. When wet conditions are presumed adhesive forces between the road surface and the car tire can be neglected, i.e. adhesion would not contribute a considerable share to the amount of friction generated. Under these conditions friction can totally be allocated to hysteretic effects within the rubber generated by the asperities of the road surface intruding into the tire tread.

Once the friction effects that are to be modeled are determined we can focus on how to model the effects. We chose to use a multi-body system containing springs and dampers and masses to model the tread rubber. Rheological elements made of springs and dampers are well accepted components for the description of visco-elastic material behavior. So we chose Maxwell elements (spring and damper in line) to describe the viscous behavior and springs to describe pure elastic behavior, see figure 3.2.

![Rheological elements used in the friction model](image)

**Figure 3.2** Rheological elements used in the friction model

3.3 Model Structure

The friction model consists of a large number of so called Poynting-Thompson elements which are interconnected by coupling springs. Each of these Poynting-Thompson elements can be considered as an “elementary” rubber element. It consists of several Maxwell elements in parallel with a spring. This arrangement assures that the stiffness of each of those “elementary” rubber elements can be expressed by equations (3.1) and (3.2).

![Structure of one “elementary” rubber element](image)

**Figure 3.3** structure of one “elementary” rubber element (Poynting-Thompson element)
4. Parameter Identification

In order to describe the behavior of one elementary rubber element like shown in figure 3.3 by equations (3.1) and (3.2) one has to identify parameters $E_0 \ldots E_n$ and $\eta_1 \ldots \eta_n$, in which $\eta_i = E_i \cdot \tau_i$. The parameter identification is conducted based on measured curves for the storage and loss modulus of a certain tire tread compound that was available to us. It is used for up-to-date summer tires. The measured curves are shown in figures 4.1 and 4.2.

![Figure 4.1 Master curve for storage modulus of modeled rubber](image1)

![Figure 4.2 Master curve for loss modulus of modeled rubber](image2)
From the curve for the storage modulus the parameters for the elastic properties of the rubber can be identified. On the other hand the curve for the loss modulus is used to identify the parameters for the viscous properties. By a method described by Emri and Tschoegl [4] 22 different elastic modules \( E_0, E_1, E_2, \ldots, E_{21} \) could be identified from the curve measured for the storage modulus. They describe the contributions to the elastic modulus at 21 associated frequencies \( \tau_0^{-1}, \tau_1^{-1}, \tau_2^{-1}, \ldots, \tau_{21}^{-1} \). The frequencies are: 0 Hz, 10^0 Hz, 10^1 Hz, 10^2 Hz, \ldots, 10^{20} \text{ Hz} \) respectively. The parameters identified fit equation (3.1) as follows:

\[
E'(\omega) = E_0 + \sum_{i=1}^{21} E_i \frac{(\tau_i \omega)^2}{1 + (\tau_i \omega)^2}
\] (4.1)

The identified 21 storage moduli along with the equilibrium modulus, \( E_0 \), are shown in figure 4.3. This kind of diagram is called a “line spectrum”.

![Line Spectrum, Storage Moduli](image)

**Figure 4.3** Line spectrum with 21 storage moduli identified from master curve shown in figure 4.1

When \( E_0 \) along with the identified 21 storage moduli \( E_1, E_2, \ldots, E_{21} \) and their respective relaxation times \( \tau_0, \tau_1, \tau_2, \ldots, \tau_{21} \) are inserted into equation (4.1) we get a very good mathematical description of the frequency-dependent storage modulus as can be seen from figure 4.4: the blue curve is the measured data, the yellow curve is equation (4.1); there is virtually no difference in the curves.

Corresponding to equation (4.1) we can write for the loss modulus (see eq. 3.2):

\[
E''(\omega) = \sum_{i=1}^{21} E_i \frac{\tau_i \omega}{1 + (\tau_i \omega)^2} = \sum_{i=1}^{21} \eta_i \frac{\omega}{1 + (\tau_i \omega)^2}
\] (4.2)

with \( \eta_i = E_i \cdot \tau_i \). Applying the same identification method (Emri-Tschoegl) to the data of the loss modulus depicted in figure 4.2 we get 21 viscosities \( \eta_1, \eta_2, \ldots, \eta_{21} \) and their respective relaxation times \( \tau_0, \tau_1, \tau_2, \ldots, \tau_{21} \) (\( \tau_i \) being the same as for the storage modulus). They are shown in figure 4.5.
Figure 4.4 measured and simulated results for the elastic modulus (storage modulus)

Figure 4.5 Line spectrum with 21 viscosities identified from master curve shown in figure 4.2
When the 21 viscosities $\eta_1, \eta_2, \ldots, \eta_{21}$ along with their respective relaxation times $\tau_1, \tau_2, \ldots, \tau_{21}$ are inserted into equation (4.2) we get a very good mathematical description of the frequency-dependent viscosity of the rubber as can be seen from figure 4.6, where the measured data (black curve) is compared to the results of eq. (4.2) which are marked by a blue curve. For measurement reasons no data could be determined for frequencies higher than $10^{10}$ Hz. But this frequency range is not needed for the rubber model anyway. Below $10^{10}$ Hz there is a good agreement between measured and simulated results.

![Figure 4.6 measured and simulated results for the elastic modulus (loss modulus)](image)

We conclude this chapter with the conclusion that the visco-elastic behaviour of rubber can well be described by equations (4.1) and (4.2) along with the parameters found ($E_0 \ldots E_{21}, \eta_1 \ldots \eta_{21}$ and $\tau_1 \ldots \tau_{21}$).

By combining the equations we get the following expression for the elastic modulus of rubber:

$$E(j \omega) = E_0 + \sum_{i=1}^{21} E_i \left(\frac{\tau_i \omega}{1 + (\tau_i \omega)^2}\right)^2 + \sum_{i=1}^{21} \eta_i \frac{\omega}{1 + (\tau_i \omega)^2}$$

(4.3)

The first expression describes the real part, the second the imaginary part of the complex modulus of elasticity.

5. The Rubber Model in Detail

As already stated in chapter 3.3 the rubber model consists of a large number of so called Poynting-Thompson elements. Each of these Poynting-Thompson elements can be considered as an “elementary” rubber element.

![Figure 5.1 “elementary” rubber element (Poynting-Thompson element)](image)
In chapter 4 we have already identified the parameters of these elements (E₀ … E₂₁, η₁ … η₂₁ and τ₁ … τ₂₁). As stated in chapter 3.3 the elementary rubber elements (ERE) are connected to one another by coupling springs. The physical structure of the rubber model is shown in figure 5.2.

![Figure 5.2](image-url)  
**Structure of the rubber model**

On the left-hand side (“front view”) of figure 5.2 we recognize one ERE. It has got an additional mass attached to it, though. The mass accounts for the mass of the tire tread. In the side view (right-hand side of figure 5.2) we see three of the EREs connected to one each other by coupling springs. The EREs in turn are connected to the tire rim by rigid joints. The unloaded length of each of them is 10 mm, which is about the height of the tire tread. The distance between them is 3.3 micrometer. The arrangement moves along the road texture profile, partly touching the road with it’s “tentacles” – partly not (see “side view”). The amount of contact, i.e. the degree of penetration of the rubber into the texture, is depending on the vertical load and the stiffness of the rubber as expressed by the EREs and the coupling springs. The rubber model can be considered as a 2D-model since it needs a 2D-input, i.e. the texture profile of the road.

The tire rim is fixed, which means that no vertical displacement of the tire takes place – which in turn means that the vertical dynamics of the wheel are neglected for the determination of friction between tire and road. Sliding between rubber and road surface is simulated by just moving the texture profile horizontally along the model structure as shown in figure 5.2.

Another important assumption is that the water between tire and road can be sufficiently drained through the cavities in the road and the channels of the tire tread. This means that any hydro-dynamic effects are excluded. A third important assumption has already been mentioned in chapter 3.2: due to the presumed water/moisture on the road the effects of adhesion between road and tire on friction can be neglected. This means we can focus on the texture shape as the only cause for hysteretic processes within the rubber; a significant contribution of adhesion to friction can be excluded.

We conclude this chapter with the statement that friction in this particular case can be assumed to be caused by texture only and corresponds to energy dissipation in the rubber – here: in the dampers of the rubber model.
6. **Determination of Spring and Damper Constants**

In order to assign specific damper and spring constants to the elements of our model (as opposed to elasticity and viscosity) we have to give it a geometrical dimension. As already mentioned in the previous chapter the height of the tread is set to 10 mm. For its length the length of an average tire contact patch has to be considered. For passenger cars this is about 130 mm. For the ease of calculation we assume a uniform wheel load distribution on a reduced contact length of 100 mm like indicated in figure 6.1.

![Figure 6.1 definition of reduced contact length](image)

The width of the contact patch is chosen to be 195 mm. Taking into account an average tread pattern we assume that the wheel load concentrates on 70% of the area of the contact patch. So, for the determination of the spring and damper constants of our model we use a reduced tread area of

\[ A = 0.7 \cdot 195mm \cdot 100mm = 13650mm^2 = 136.5cm^2 \]

Since the EREs are spaced with a distance of 3.3 micrometers we end up with 30,000 EREs over the whole length (= reduced contact length) of 100 mm.

So, each of the EREs is assigned a contact area of

\[ \Delta A = 13,650mm^2 / 30,000 = 0.455mm^2 \]

In figure 6.2 a simple formula is given by which the spring and damper constants can be determined from elasticity \( E_i \) and viscosity \( \eta_i \) with known area \( \Delta A \) and height \( H \).

\[
\begin{align*}
\text{spring constant } c_i & = \frac{\Delta A}{H} \cdot E_i \\
\text{damper constant } k_i & = \frac{\Delta A}{H} \cdot \eta_i
\end{align*}
\]

![Figure 6.2 calculation of spring and damper constant from storage modulus and viscosity respectively](image)
7. Including Strain and Temperature Dependency

Now, that we have identified 22 frequency-dependent spring and 21 frequency-dependent damper constants from $E_0, E_1, \ldots, E_{22}$ and $\eta_1, \ldots, \eta_{21}$ via equations (6.1) given in figure 6.2 we go over to include the strain and temperature-dependent behaviour as described in chapter 2. Both influences can be included into the model by making the spring and damper constants temperature- and strain-dependent.

7.1 Including Strain Dependency

By using the model developed by Kraus [5] we can formulate strain-dependent spring and damper constants as follows:

$$c_{i,e} = \frac{K_c}{1 + \left(\frac{\varepsilon}{\varepsilon_c}\right)^{2m_1}} \cdot c_i \quad \text{and} \quad k_{i,e} = \frac{K_k \cdot \left(\frac{\varepsilon}{\varepsilon_c}\right)^{m_2}}{1 + \left(\frac{\varepsilon}{\varepsilon_c}\right)^{2m_2}} \cdot k_i \quad (7.1)$$

We see the strain incorporated in equations (7.1) in the form of a factor $(\varepsilon/\varepsilon_c)^m$. From experiments the following constants could be identified:

- $K_c = 1.83$
- $K_k = 2.03$
- $\varepsilon_c = 0.3$ (reference strain)
- $m_1 = 0.23$
- $m_2 = 0.45$

Figure 7.1 shows the results of measurements for the storage modulus carried out with an excitation frequency of 10 Hz along with theoretical results obtained by using equations (7.1) and (6.1). We can see a very good agreement between experimental and fitted theoretical results.

![Figure 7.1 fitting Kraus model to measured data of storage modulus](image.png)
In the same way the Kraus model has been fitted to data measured for the loss modulus. The results can be seen in figure 7.2. We conclude this chapter by stating that the strain dependency can well be expressed by equations (7.1) along with the constants given above.

![Figure 7.2 fitting Kraus model to measured data of loss modulus](image)

### 7.2 Including Temperature Dependency

As already mentioned in chapter 2 a temperature increase results in a frequency shift of the curves for the storage and loss modulus towards higher frequencies resulting in a decrease in stiffness. The shift factor is calculated according to William, Landel and Ferry by the “WLF” equation (7.3). But as a preliminary step the temperature increase has to be determined as a result of the dissipated energy within a considered time step. It is depending on the volume and the specific heat capacity of the considered rubber element:

\[
\Delta T = \frac{W_{\text{diss, } \Delta t}}{C_s \cdot V} \tag{7.2}
\]

with $W_{\text{diss, } \Delta t} :=$ dissipated energy within a considered time step

$C_s :=$ specific heat capacity of the rubber

$V :=$ volume of the considered rubber element

For the adiabatic case a specific heat capacity of $C_s = 1 \text{ Nmm/(mm}^3 \text{ K)}$ can be set for rubber.

Once the current temperature is determined for the rubber element in consideration the shift factor can be calculated via WLF equation:

\[
\log(a_f) = \frac{C_1 \cdot (T - T_{\text{ref}})}{C_2 + (T - T_{\text{ref}})} \tag{7.3}
\]
C₁ and C₂ are constants (C₁ = -17.44; C₂ = 51.6) and T_{ref} is the reference temperature for which the master curves for storage and loss modulus have been defined.

With the thusly determined shift factors we get the following expressions for the spring and damper constants:

\[
\begin{align*}
c_{i,T} &= a_T \cdot c_{i,T_{ref}} \\
k_{i,T} &= a_T \cdot k_{i,T_{ref}}
\end{align*}
\]  \tag{7.4}

with

\begin{align*}
c_{i,T} &:= \text{spring constant of spring } i \text{ at temperature } T \\
k_{i,T} &:= \text{damper constant of spring } i \text{ at temperature } T
\end{align*}

With the fundamentals presented in chapters 1 to 7 we are now able to identify and describe the frequency-, strain- and temperature-dependent behaviour of rubber by a rheological model made of spring and dampers.

8. Systems Equations

Input parameter of the system is the exciting texture shape w(t) as shown in figure 8.1, right-hand side (“side view”). Output parameter is the deformed rubber shape h(t) as well as the “inner” displacements z_i(t) of each of the elementary rubber elements (ERE), see left-hand side of figure 8.1. The number of variables z_i is depending on the number of Maxwell (= spring-damper in line) elements. According to chapter 4 a total number of 21 Maxwell elements would be necessary to describe the elastic modulus up to a frequency of 10^{20} Hz.

![Figure 8.1 rubber system to be modeled with input and output parameters](image-url)
Figure 8.2 shows the differential equation system. This equation system must be solved for any given time step. Only the first 2 elementary rubber elements (ERE) can be illustrated (from a total number of up to 30,000). To the left of the equation we find the first derivative of the solution vector, which contains (top-down) the “inner” displacements \(z_1 \ldots z_n\), the (outer) displacement \(h\) and velocity \(\dot{h}\) of the first 2 EREs. On the right-hand side of the equation we find to the very right the excitation vector, \(w_i^*(t)\), mainly influenced by the texture shape \(w(t)\) which is moved horizontally underneath the rubber system with the given sliding velocity. Besides that the mass of the ERE, \(m\), and the stiffness of the road surface determines \(w_i^*(t)\): 

\[ w_i^*(t) = \frac{c_{\text{asp}}}{m} \cdot w_i(t) \]  

(8.2)

To the left of the excitation vector, \(w_i^*(t)\), we find the solution vector, and left of that the huge sparse matrix which is built up by elements containing the masses, spring and damper constants.

with 

\( c_{ij} := \) spring constant of spring i in ERE j  
\( c_{kj} := \) spring constant of coupling spring of ERE j  
\( k_{ij} := \) damper constant of damper i in ERE j  
\( m := \) mass of ERE  
\( \Sigma c_{ij} := \) abbreviation for \( c_{0j} + 2c_{kj} + \Sigma c_{ij} + c_{\text{asp}} \)  

with 

\( c_{0j} := \) spring constant according to \( E_0 \) of ERE j (see figure 3.3)  
\( c_{\text{asp}} := \) spring constant of road surface
The above mentioned matrix elements (except for the mass of course) are frequency-, strain- and temperature-dependent.

If the counting variable \( i \) is limited to 1:4, i.e. if the number of Maxwell elements per ERE is limited to 4 (see figure 3.3), we get a sparse matrix with entries according to figure 8.3. The left side shows only a part of the sparse matrix exhibiting 5 EREs. The whole sparse matrix is displayed on the right-hand side of figure 8.3.

![Figure 8.3](image)

**Figure 8.3** the sparse matrix of the differential equation system in detail (left) and overview (right) (EREs with 4 Maxwell elements only)

9. **Program Structure**

The program consists of a main program in which the mechanical system is defined and subprograms in which the time step analysis is performed, i.e. in which the differential system is set up and solved.

The following tasks are done in the main program:

- Defining the physical system (number and size of elements)
- Defining the material properties (spring/damper constants, temperature and strain behaviour)
- Defining the degrees of freedom and the systems excitation
- Defining the initial conditions of the system
- Calling the subprogram in which the time step analysis is done (calling the file in which the differential equation system is defined)
- Invoking the ODE (ordinary differential equation) solver
The following tasks are done in the subprogram (ODE file):

- Defining the given contact conditions of the system at the specified time
- Calculating the input excitation and resulting forces
- Calculating the strain and temperature influence at the specified time and location
- Calculating the elements of the sparse matrix
- Assembling the sparse matrix and defining the ODE system
- Calculating the dissipated energy and temperature rise at the specified time and location
- Calculating the coefficient of friction at the specified time and location

10. Program Modules

There are 2 basic modules to which the above mentioned structure is applied:

- The penetration depth module, and
- The friction module

Both modules are using the same program structure and program algorithm. The only difference is the kind of movement; the penetration depth module calculates the vertical movement of the rubber tread into the texture, whereas the friction module calculates the horizontal (sliding) movement of the rubber tread along the road. Both modules shall be explained in the following chapters.

10.1 Penetration Depth Module

The penetration depth module has got the task to determine to which amount the rubber tread penetrates into the surface texture of the road. This information is crucial for the rubber model because the penetration depth determines the resulting texture profile that the tire faces and thus the excitation function of the rubber.

Contact time: \( T = \frac{L}{v_{\text{wheel}}} \)

Loading: Half Sine

Frequency: \( \omega = \frac{2\pi v_{\text{wheel}}}{(2L)} \)

Loading function: \( p = mg \sin \omega t \)

Figure 10.1 penetration model
A tread element when passing through the tire-road contact area faces a sinusoidal loading. The contact time of the tread element is depending on the velocity of the wheel which, for a freely rolling wheel, is equivalent to the velocity of the car. It can be written: \( T = \frac{L}{v_{\text{wheel}}} \) with \( L \) being the length of the contact area. The loading function is a half-sine (see figure 10.1). The loading frequency is \( \omega = \frac{2\pi \cdot v_{\text{wheel}}}{2L} \) and the loading function thus can be assumed to: \( f(t) = 0.25 \cdot m_{\text{car}} \cdot g \cdot \sin \omega t \).

Basic idea of the penetration model is that a peace of rubber having the length and width of the contact area is vertically brought into contact with the road surface represented by the texture profile (see figure 10.1). The loading is sinusoidal with the above mentioned loading data. This movement is calculated by a time step analysis using the rubber model, the rubber parameters and program described in the previous chapters. Figure 10.2 shows the results of penetration depths for different sliding velocities. It can clearly be seen that the penetration depth decreases with increasing speed of the car, which is obvious since with increasing speed i.e. exciting frequency the rubber becomes stiffer and thus penetrates the road surface to a lower extent.

![Figure 10.2](image)

**Figure 10.2** penetration depth as a function of speed as calculated by the penetration model

The penetration depths vary between 1.08 and 0.89 mm for a speed range of 0 to 100 km/h. Figure 10.3 puts the results in a context with penetration depths found in the literature [6] (yellow curves for 3 normal and 1 coarse asphalt). Based on the measured texture profiles of 3 different asphalt surfaces - one coarse stone mastix asphalt 0/11 (blue line), one coarse asphalt concrete 0/11S (red line) and 8 normal asphalt concrete surfaces 0/8S (grey shaded area) - the appropriate penetration depths were calculated for different wheel velocities. It can be seen that the results found with the penetration model well fit with the results found in literature. The decrease found in literature is steeper though. But we are not at the end yet with the development of the rubber model. So there will be some more improvement in this respect.
Figure 10.3  penetration depths for varying speed – comparison with results found in the literature

Figure 10.4 finally shows the deformed shape of the rubber at the end of the penetration process. In the enlargement you can see how the rubber “flows” around the asperities of the texture.

Figure 10.4  rubber deformation due to vertical penetration of the rubber tread into the texture
10.2 Friction Module

Once the maximum penetration depth is determined the friction during horizontal sliding can be calculated, i.e. the final conditions of the penetration process are the initial conditions of the horizontal sliding process from which the coefficient of friction shall be calculated.

Figure 10.5 friction model

The basic assumptions for friction calculation are visualized in figure 10.5: the tread rubber has reached its maximum penetration depth and the tread elements, i.e. the interconnected elementary rubber elements (ERE) are moved horizontally with an assumed slip speed over the surface profile. The length of contact is 100 mm according to figure 6.1.

The same procedure as described in connection with the penetration module is now being applied to the friction module: the horizontal movement is calculated by a time step analysis using the rubber model, the rubber parameters and program described in the previous chapters. Figure 10.6 shows a first result of friction calculation using 24 time steps. After about 10 time steps a friction coefficient of about 0.5 and 0.9 appears. The high friction values in the beginning are caused by incorrect initial conditions. Although the friction module has to be overworked numerically the results shown in figure 10.6 are promising.

Figure 10.6
first results of friction calculation
The next figure, 10.7, takes a close look to the interface between rubber and texture during sliding. We can see the rubber sliding over the pavement texture in the indicated direction of motion. While the vertical movement generates “symmetric” shapes of the rubber flowing over the asperities (see figure 10.4) we watch unsymmetric flow shapes in this case of horizontal sliding: the rubber follows the contour very tightly at the front of the asperities and tears off on their back.

![Figure 10.7 rubber deformation due to horizontal sliding of the rubber over the texture](image)

**Figure 10.7** rubber deformation due to horizontal sliding of the rubber over the texture

### 11. State of the Model and Next Steps

At the very moment we are facing problems with the horizontal movements. The rubber is not flowing correctly over the asperities as can be seen in figure 10.7: in some places it penetrates the pavement where it is not supposed to do so. We are searching for the reasons. Maybe the required vertical movements are much too steep with respect to the horizontal. There might be a problem of discretization. Both our self-programmed solvers and the MATLAB solvers fail.

Once the problem is solved we need to limit the number of elementary rubber elements (ERE) to a minimum required number in order to save computation time. For the same reason we need to install a mechanism that invokes only the relevant Maxwell elements depending on the sliding speed.

The next step is to validate the model by means of the experimental results that we have got. We have to prove whether we get reliable results concerning the penetration depth as well as concerning the friction process.
12. References


