Provably Safe Maneuvers of Automated Vehicles

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EU Project UnCoVerCPS: Partners

Unifying Control and Verification of Cyber-Physical Systems
(UnCoVerCPS)

Funding: 4.9 mio Euro

Participant organisation name	Country
Technische Universität München (TUM)	Germany
Université Joseph Fourier Grenoble 1 (UJF)	France
Universität Kassel (UKS)	Germany
Politecnico di Milano (PoliMi)	Italy
GE Global Research Europe (GE)	Germany
Robert Bosch GmbH (Bosch)	Germany
Esterel Technologies (ET)	France
Deutsches Zentrum für Luft- und Raumfahrt (DLR)	Germany
Tecnalia (Tec)	Spain
R.U.Robots Limited (RUR)	United Kingdom

EU Project UnCoVerCPS: Main objectives

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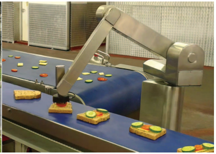
- A new development process that reduces development time and costs for critical cyber-physical systems.

Focus Of This Talk: Online Verification of Automated Cars

Thought experiment

How many possible situations is an automated car facing?

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- Infinitely many!

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How many possible situations is an automated car facing?

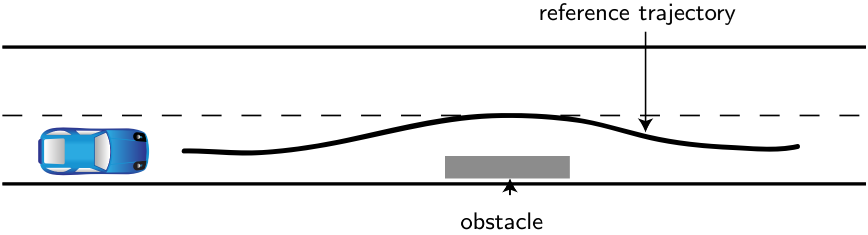
- Infinitely many!
- O.k., let's discretize (see details below): At least 10^{81} (outnumbers the estimated amount of atoms in the universe)

Problem: How should this be verified upfront?
Solution: We have to verify the vehicle while it is in operation
$\rightarrow$ online verification.

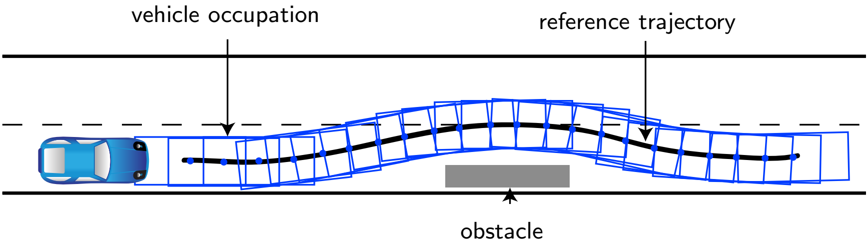
Discretization of the problem:

- Each surrounding vehicle: position (x- and y-coordinate), velocity and orientation (4 variables).
- Each lane: width, curvature, and change of curvature (3 variables).
- Own vehicle: x- and y-coordinate, velocity, orientation, yaw rate, steering angle (6 state variables) and tire-road friction, current loading (2 variables).
- Bounds on numbers of variable values and objects: 20 values per variable, maximum 10 surrounding vehicles, 5 lanes.
- Result: $(20^4)^{10} \cdot (20^3)^5 \cdot 20^6 \cdot 20^2 \approx 10^{81}$.

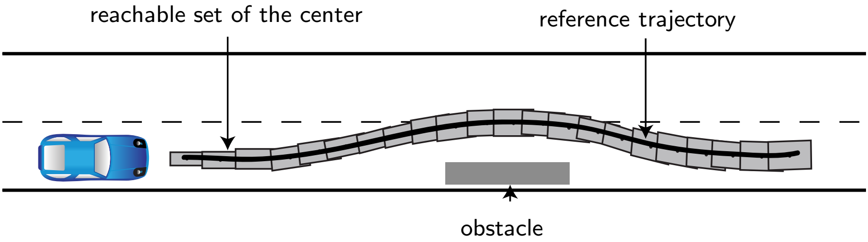
Trajectory Verification: Situation



Trajectory Verification: Standard Approach



Trajectory Verification: Considering Uncertainties



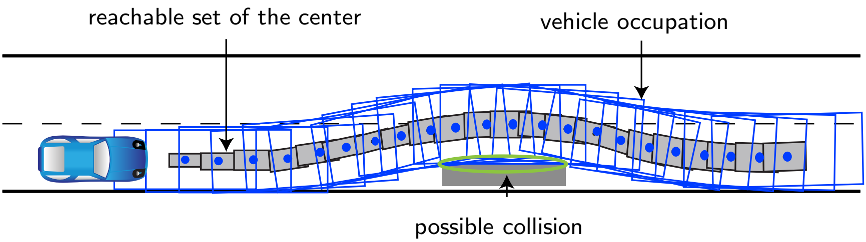
Robust Safety Problem

Is the planned maneuver of the autonomous vehicle still safe under

- uncertain initial states,
- uncertain measurements,
- and disturbances?

Objective: Guarantee safety when bounds on uncertainties are known.

Trajectory Verification: Formal Verification Reveals Problems



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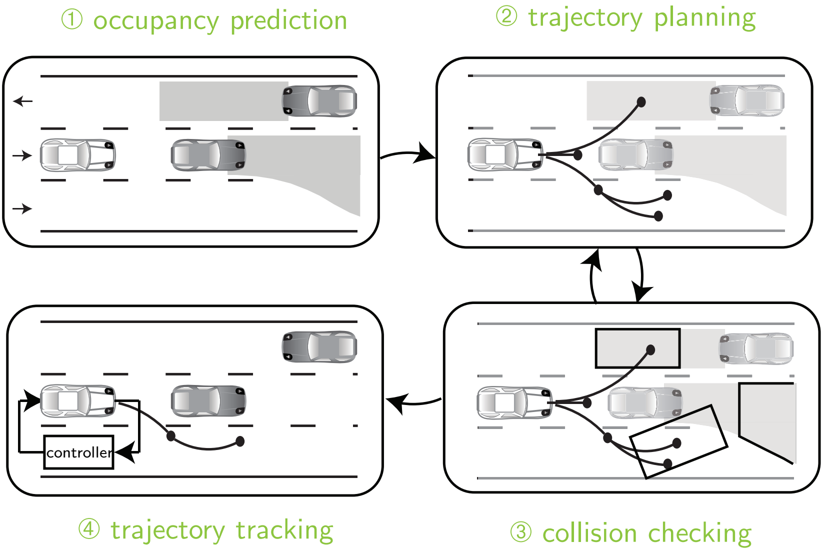
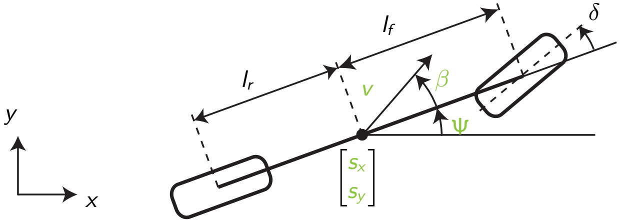
Outline

1. Overview of the approach
2. Models of the ego vehicle and other traffic participants
3. Verification procedure
4. Test results
5. Verification of high-fidelity vehicle models
6. Pre-computation using motion primitives
7. Adaptation for automated driving

Overview of the Approach



Model of the Uncontrolled Ego Vehicle



$$\dot{\beta} = \left(\frac{C_r l_r - C_f l_f}{m v^2} - 1\right)\dot{\Psi} + \frac{1}{m v}\left(C_f \delta - (C_f + C_r)\beta\right)$$
$$\dot{\Psi} = \dot{\Psi}$$
$$\ddot{\Psi} = \frac{1}{I_z}\left((l_r C_r - l_f C_f)\beta - (l_f^2 C_f + l_r^2 C_r)\frac{\dot{\Psi}}{v} + l_f C_f \delta\right)$$

yaw dynamics

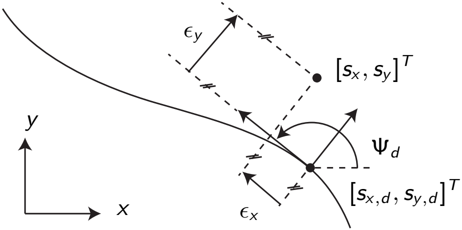
$$\dot{v} = a_x$$

longitudinal dynamics

$$\dot{s}_x = v \cos(\beta + \Psi)$$
$$\dot{s}_y = v \sin(\beta + \Psi)$$

position of the vehicle

Tracking Controller



$$\delta = k_1(\epsilon_y + u_{\epsilon_y}) + k_2(\Psi_d - \Psi - u_\Psi) + k_3(\dot{\Psi}_d - \dot{\Psi} - u_{\dot{\Psi}}) \quad \text{steering control}$$

$$a_x = k_4(\epsilon_x + u_{\epsilon_x}) + k_5(v_d - v - u_v) \quad \text{longitudinal control}$$

- Reference values: Ψ_d, $\dot{\Psi}_d$, v_d.
- Sensor noises: u_{ϵ_x}, u_{ϵ_y}, u_Ψ, $u_{\dot{\Psi}}$, u_v.
- Combining the vehicle model and the control laws yields the final model.

Constraints for Traffic Participants

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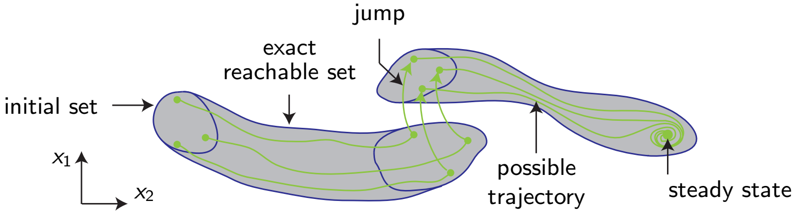
When a violation of a constraint of a traffic participant is sensed, it is no longer considered in future predictions for that particular traffic participant.

Reachability Analysis

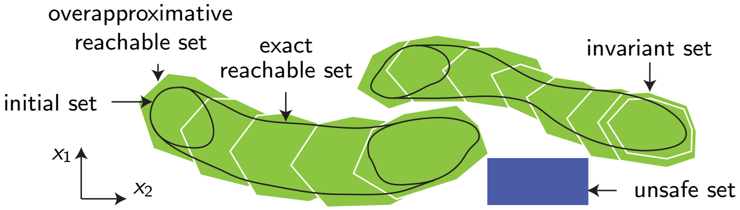


Informal Definition

A reachable set is the set of states that can be reached by a dynamical system in finite or infinite time for a

- set of initial states,
- uncertain inputs,
- and uncertain parameters.

Overapproximative Reachable Sets

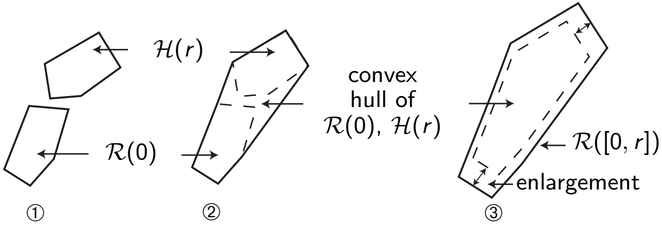


- Exact reachable set only for special classes computable
  $\rightarrow$ overapproximation computed for consecutive time intervals.
- Overapproximation might lead to spurious counterexamples.
- Simulation cannot prove correctness.

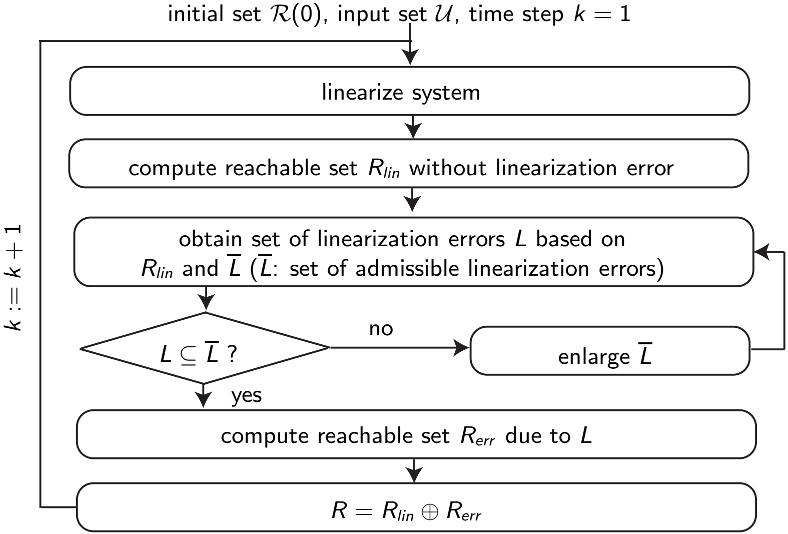
Linear Systems: Overview of Reachable Set Computation

$\dot{x}(t) = Ax(t) + u(t), \quad A \in \mathbb{R}^{n\times n}, \quad x(t) \in \mathbb{R}^n, \quad x(0) \in \mathcal{R}(0), \quad u(t) \in u_c \oplus \mathcal{U}$

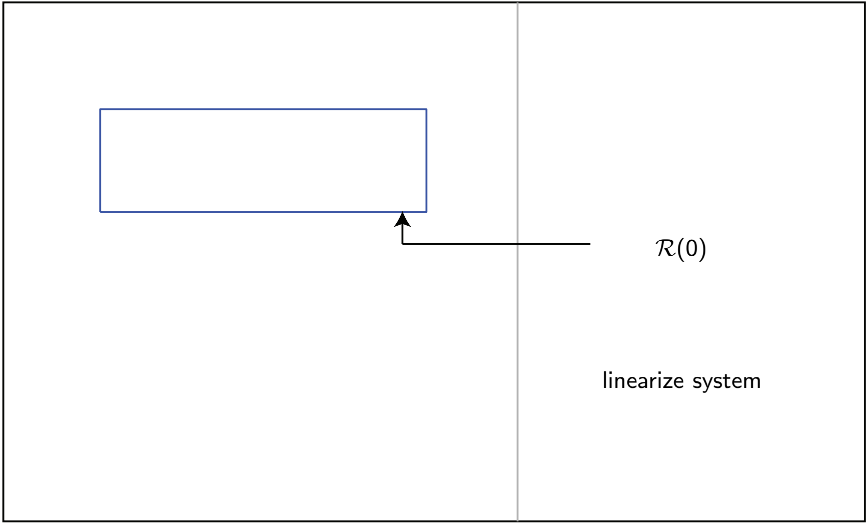
1. Compute reachable set $\mathcal{H}(r) = e^{Ar}\mathcal{R}(0) \oplus \int_{t=0}^{r} e^{A(r-t)}\mathrm{d}t\, u_c$ at time r neglecting the uncertain input ($\mathcal{C} \oplus \mathcal{D} := \{c + d | c \in \mathcal{C}, d \in \mathcal{D}\}$).
2. Obtain convex hull of initial set $\mathcal{R}(0)$ and $\mathcal{H}(r)$.
3. Enlarge reachable set to account for (1) uncertain inputs, (2) curvature of trajectories.
4. Continue with further time intervals $[kr, (k+1)r]$, $k \in \mathbb{N}$.



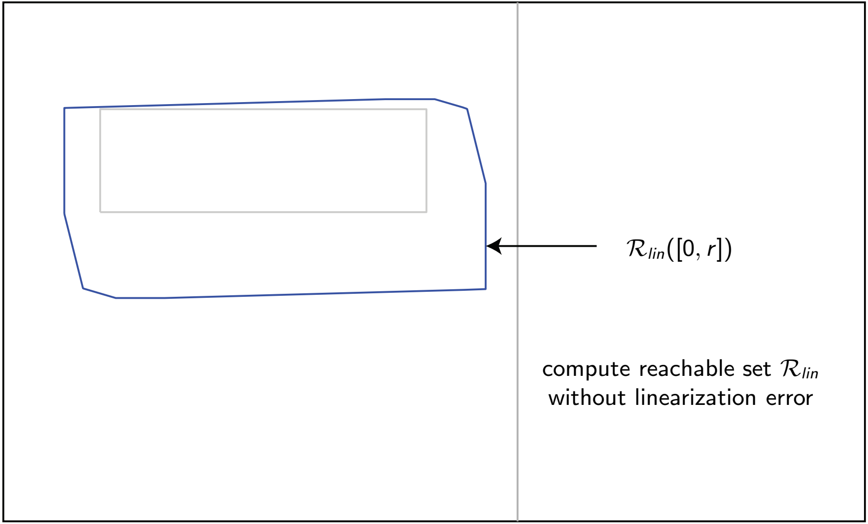
Nonlinear Reachability Analysis: Overall Algorithm



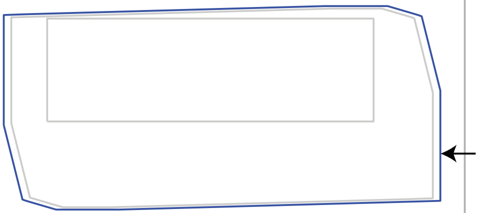
Overall Algorithm: Animation (I)



Overall Algorithm: Animation (II)



Overall Algorithm: Animation (III)



$\mathcal{R}_{lin}([0, r]) \oplus \overline{\mathcal{R}}_{err}([0, r])$

$\overline{\mathcal{R}}_{err}$: reachable set due to $\overline{\mathcal{L}}$

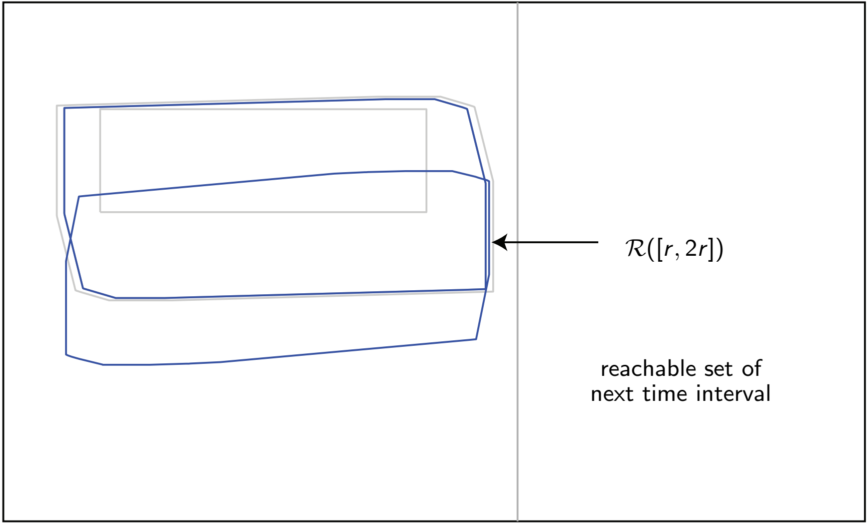
obtain set of linearization errors $\mathcal{L}$ based on $\mathcal{R}_{lin}([0, r]) \oplus \overline{\mathcal{R}}_{err}([0, r])$

Overall Algorithm: Animation (IV)

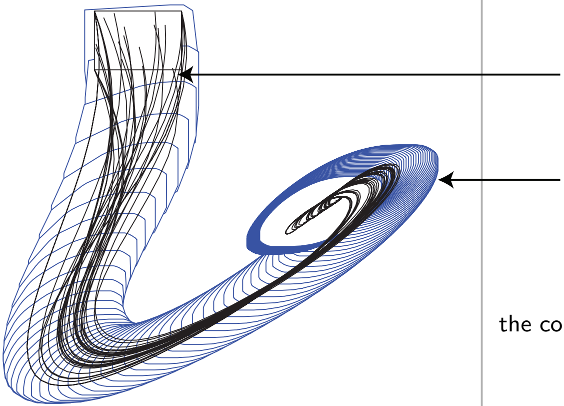
$$\mathcal{R}([0,r]) = \mathcal{R}_{lin}([0,r]) \oplus \mathcal{R}_{err}([0,r])$$

$$\mathcal{L} \subseteq \overline{\mathcal{L}} \text{ ?}$$

Overall Algorithm: Animation (V)



Overall Algorithm: Animation (VI)

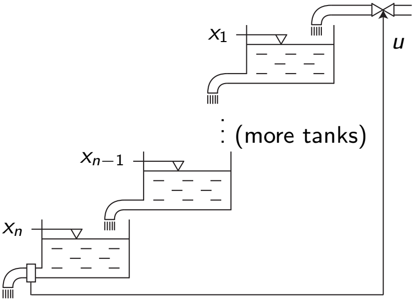


possible trajectories

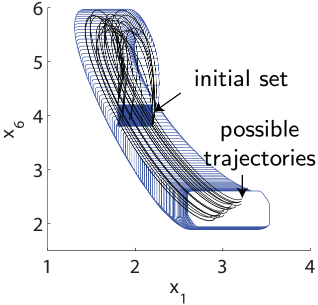
$\mathcal{R}([0, t_f])$

reachable set of the complete time horizon t_f

Scalability of the Linearization Approach



Water tank system.

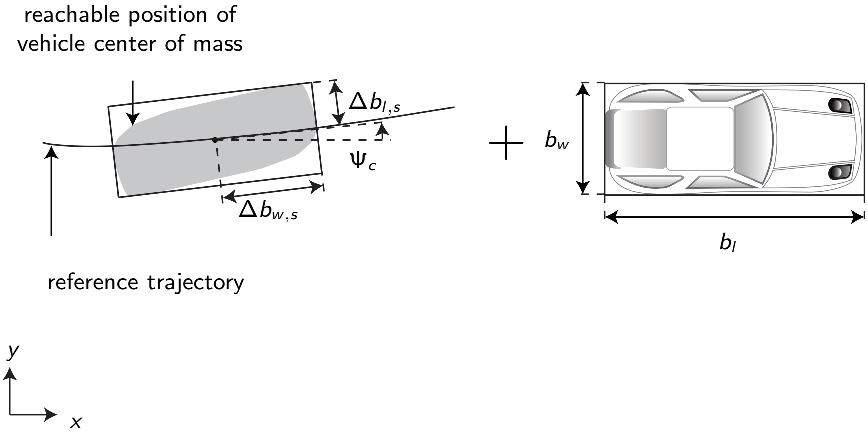


Projected reachable set ($n = 6$).

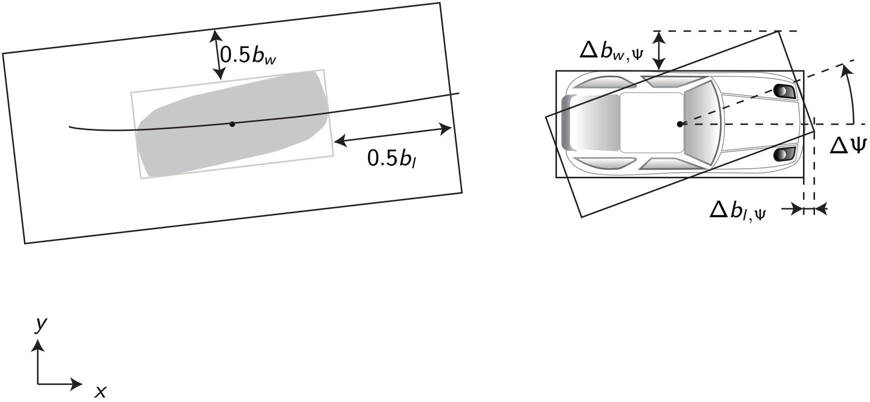
Complexity with respect to the number of continuous state variables n: $\mathcal{O}(n^3)$.

Dimension n	5	10	20	50	100
CPU-time [sec]	1.19	1.73	3.11	11.59	35.78

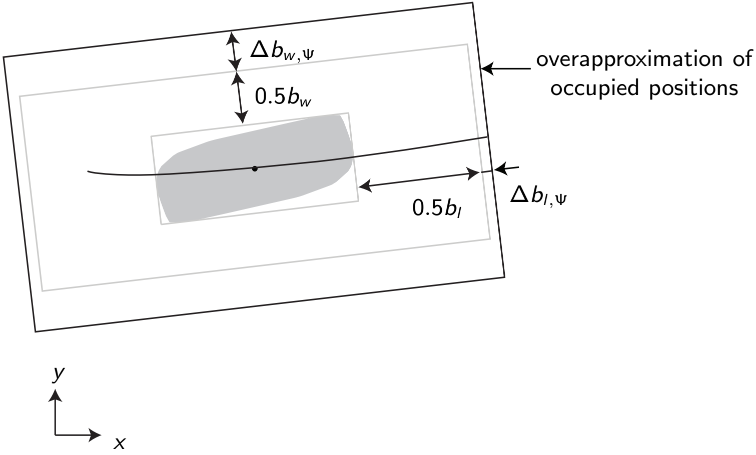
Occupied Positions: Step 1



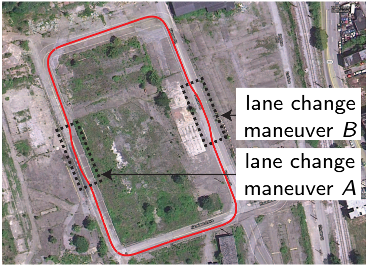
Occupied Positions: Step 2



Occupied Positions: Step 3

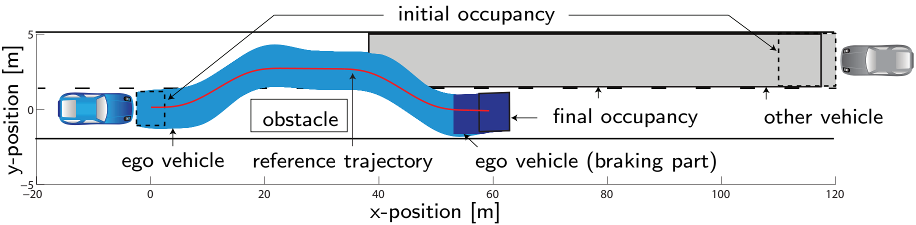


Online Verification Of Automated Driving

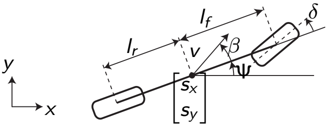


Test site

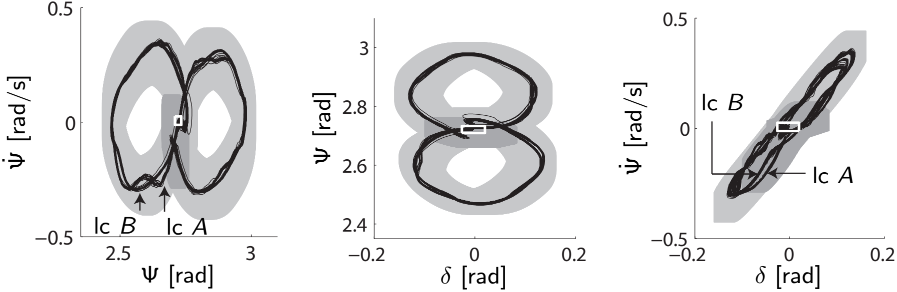
Test vehicle



Test Drive Results



s_x, s_y [m]	x- and y-position
Ψ [rad]	orientation
β [rad]	slip angle at center of mass
δ [rad]	front wheel angle
v [m/s]	velocity



computation time: $\approx$ 1.8 times faster than maneuver time (Intel i7, 1.6GHz)

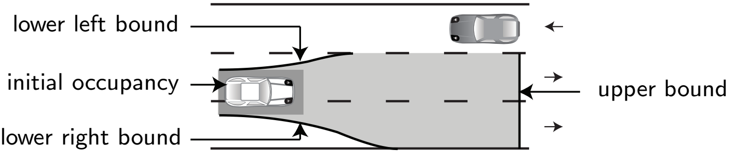
Abstraction Technique for Other Traffic Participants

Overapproximative Occupancy

Given are models M_i, $i = 1 \dots m$ which are abstractions of model M_0, i.e., $\mathrm{reach}(M_0) \subseteq \mathrm{reach}(M_i)$. The occupancy of the model M_0 can be overapproximated by

$$\mathrm{proj}\big(\mathrm{reach}(M_0)\big) \subseteq \bigcap_{i=1}^{m} \mathrm{proj}\big(\mathrm{reach}(M_i)\big). \qquad \square$$

Two models: Longitudinal dynamics along road boundaries (upper bound), lateral dynamics towards road boundaries (left/right bound).



Occupancy Along Road Boundaries

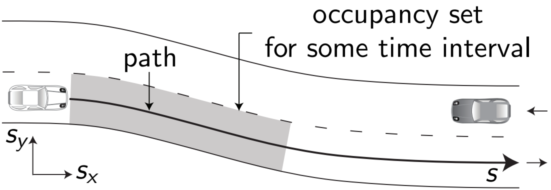
The dynamics becomes monotone when following a lane center.

Definition (Monotone dynamics)

For the initial state $x(0) \in \mathcal{R}(0)$ and inputs $u(t) \in \mathcal{U}$ the dynamics is monotone when the following holds for the solution $\chi(t, x(0), u(\cdot))$:

$$\text{if } \forall i, j, t \geq 0 : x_i(0) \leq \bar{x}_i(0), \quad u_j(t) \leq \bar{u}_j(t) \text{ then}$$
$$\forall i, t \geq 0 : \chi_i(t, x(0), u(\cdot)) \leq \chi_i(t, \bar{x}(0), \bar{u}(\cdot)). \qquad \square$$

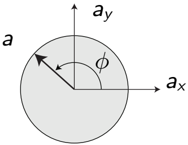
From this follows that e.g. the upper bound is provided by max. position, max. velocity, and max. acceleration:

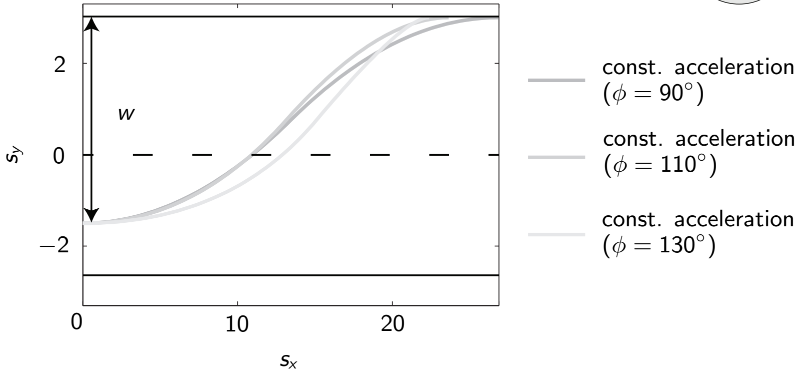


Occupancy Towards Road Boundaries

For lateral dynamics there exists no single combination of an initial state and an input trajectory determining the boundary.

Given the vehicle-fixed angle of the acceleration vector a, possible trajectories are:





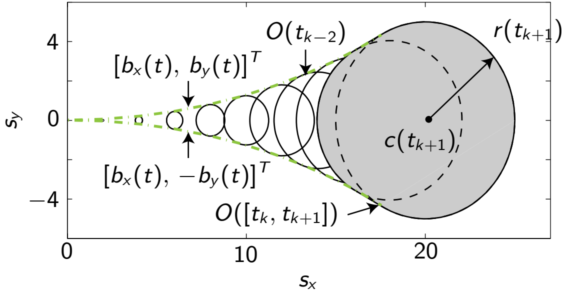
Occupancy Towards Road Boundaries: Method A

Using limit of absolute acceleration (constraint $C4$): Occupancies are circles with center $c(t)$ and radius $r(t)$:

$$c(t) = \begin{bmatrix} s_x(0) \\ s_y(0) \end{bmatrix} + \begin{bmatrix} v_x(0) \\ v_y(0) \end{bmatrix} t, \quad r(t) = \frac{1}{2} a_{\max} t^2.$$

From this follows the boundary of occupation:

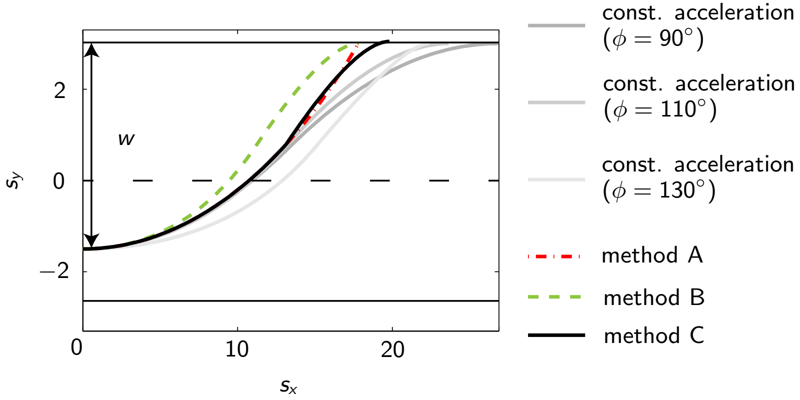
$$b_x(t) = v_0 t - \frac{a_{\max}^2 t^3}{2 v_0}, \quad b_y(t) = \sqrt{\frac{1}{4} a_{\max}^2 t^4 - \left(\frac{a_{\max}^2 t^3}{2 v_0} \right)^2}.$$



Occupancy Towards Road Boundaries: Method B and C

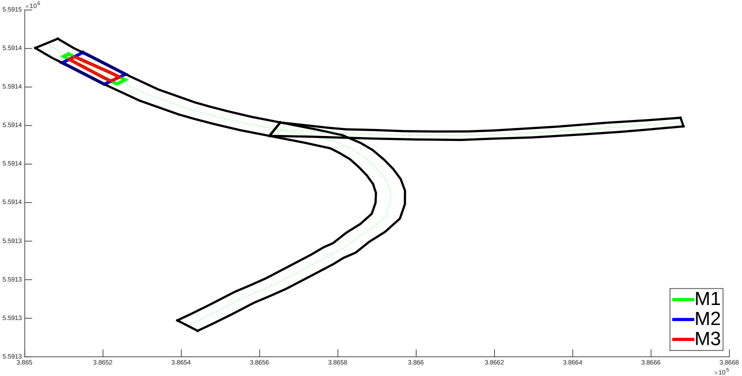
Method B: Assume independence of lateral and longitudinal acceleration $\rightarrow$ analytical solution.

Method C: Combination of method A and B.



Examples: Lane 1

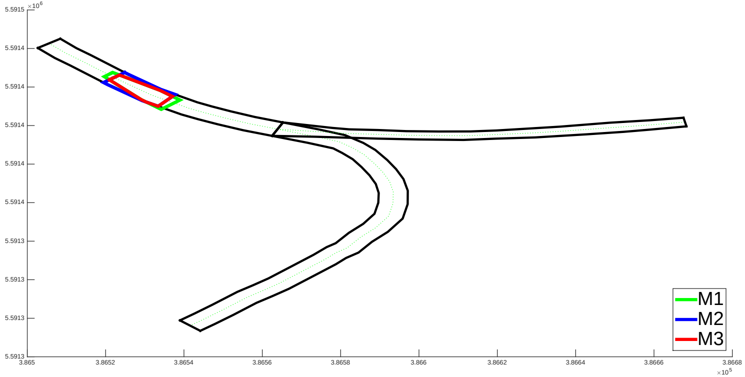
Step 1:



- M1: restricted absolute acceleration.
- M2: restricted acceleration and velocity in longitudinal direction.
- M3: staying within road boundaries.

Examples: Lane 1

Step 2:



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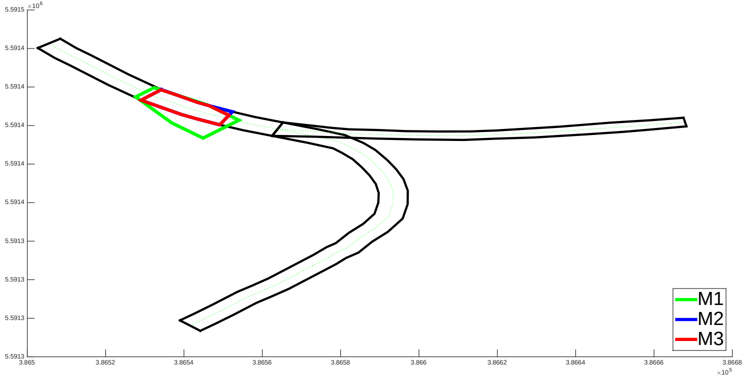
Examples: Lane 1

Step 3:



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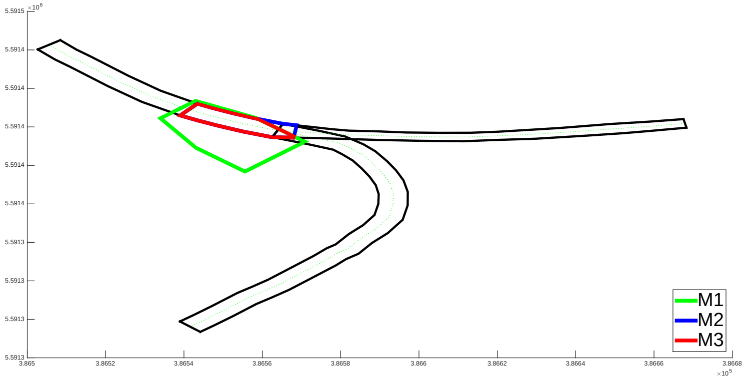
Examples: Lane 1

Step 4:



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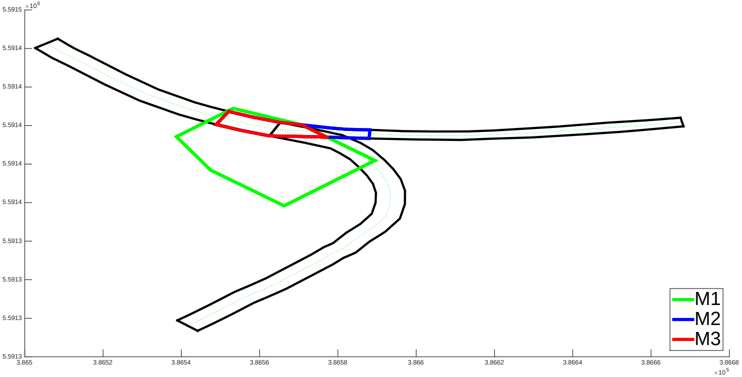
Examples: Lane 1

Step 5:



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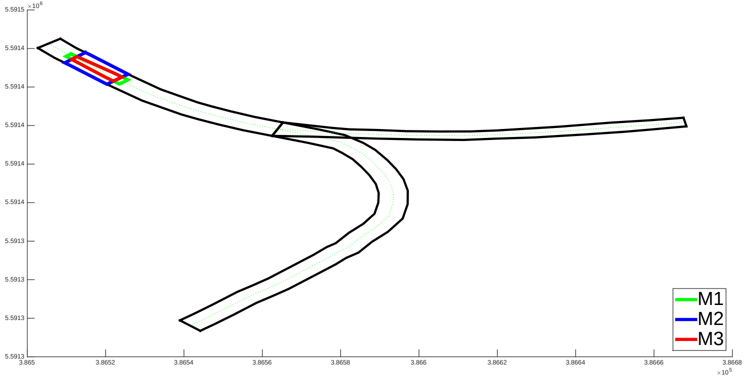
Examples: Lane 2

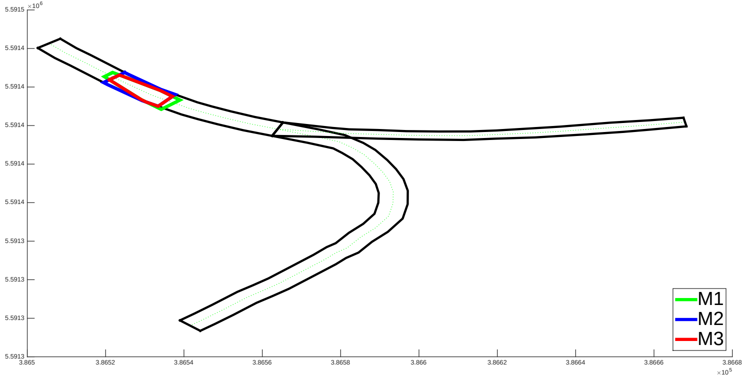
Step 1:



- M1: restricted absolute acceleration.
- M2: restricted acceleration and velocity in longitudinal direction.
- M3: staying within road boundaries.

Examples: Lane 2

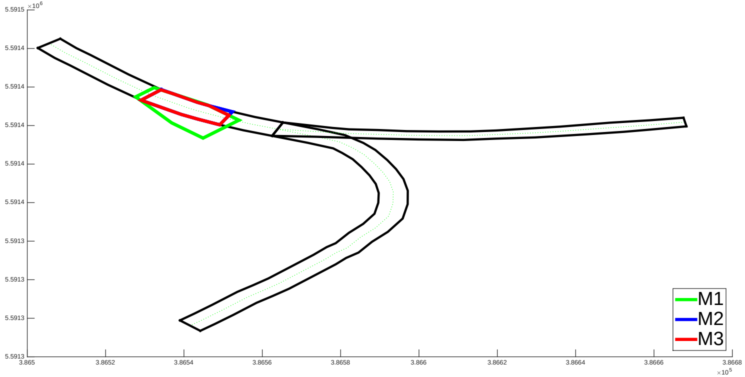
Step 2:



- M1: restricted absolute acceleration.
- M2: restricted acceleration and velocity in longitudinal direction.
- M3: staying within road boundaries.

Examples: Lane 2

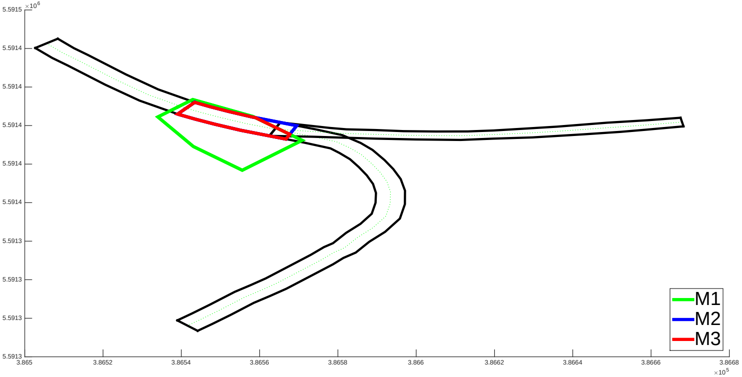
Step 3:



- M1: restricted absolute acceleration.
- M2: restricted acceleration and velocity in longitudinal direction.
- M3: staying within road boundaries.

Examples: Lane 2

Step 4:



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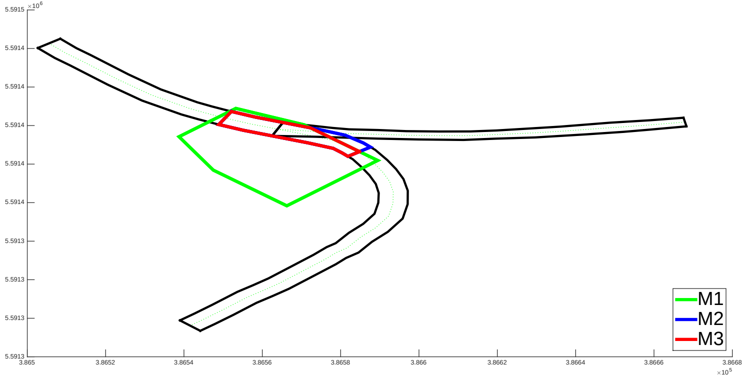
Examples: Lane 2

Step 5:



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Methods for High-Order Models

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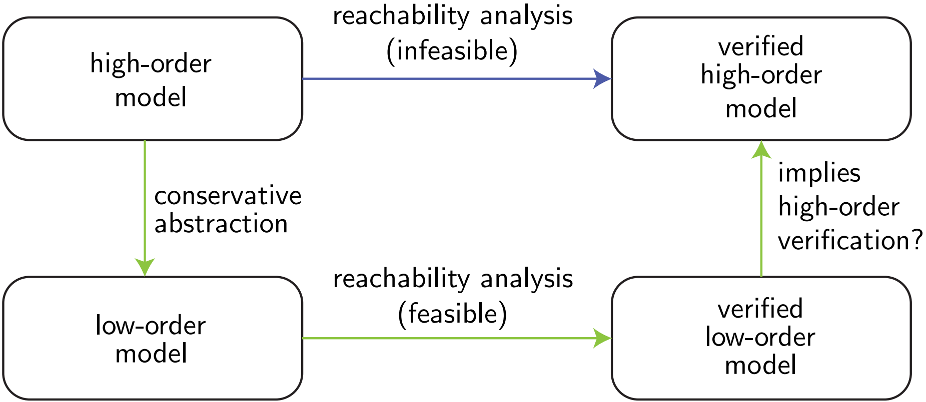
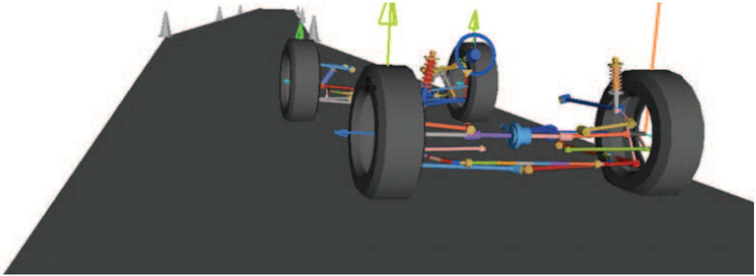
$\rightarrow$ Falsification of the low order model.

High-Order Model Verification Using Low-Order Models

Conservative abstraction (include uncertainties):



High-Order Vehicle Model



Source: `www.bremarauto.com`

Features:

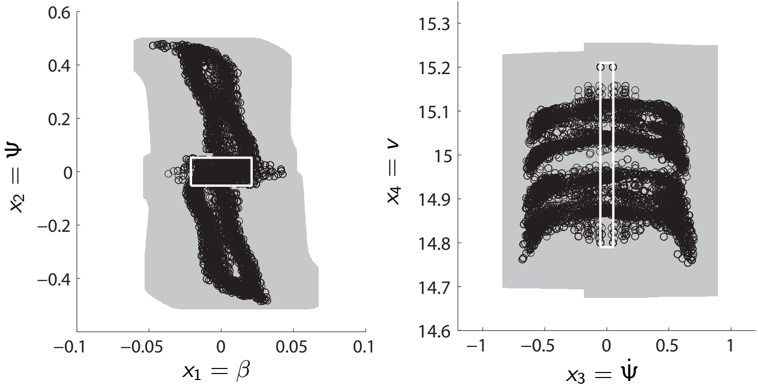
- Multi-body dynamics (28 state variables)
- Individual tire spin, slip, and camber angle.
- Nonlinear tire dynamics according to PAC2002 Magic-Formula.
- Suspension forces from springs, dampers, and anti-roll bars.

Falsification of Low-Order Models

Choose a finite number of maneuvers. For each maneuver:

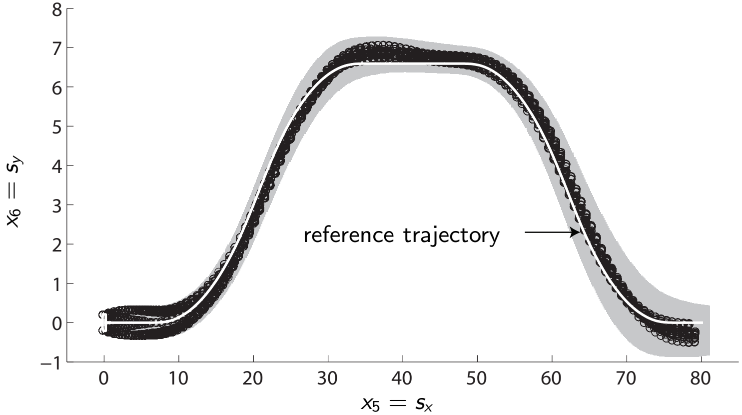
1. Compute the reachable set of the bicycle model using
$\dot{x} = f(x(t), x_d(t), u(t)) + v(t)$,
(x: state, x_d: desired state, u: sensor noise, v: additional disturbance).
2. Use rapidly-exploring random trees (RRTs) to guide the simulation of the high-fidelity model outside the reachable set.
3. In case of a violation, increase the uncertain input set $v(t) \in \mathcal{V}$ and go back to step 1.

Double Lane Change Test (1)



- The white set shows the set of initial states.
- Black circles show RRT nodes of the high-dimensional model.
- Gray area shows reachable set of the low-dimensional model.

Double Lane Change Test (2)



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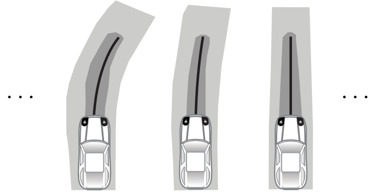
Connecting Reachability Results of Motion Primitives

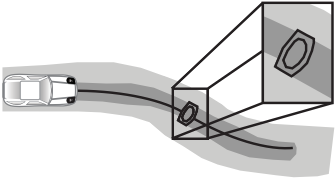
Basic idea: Pre-compute reachable sets of motion primitives.

Connection constraint: Results can be combined when final set is a subset of the initial set of the next motion primitive.

Result: Computational effort is shifted towards offline computation.



Reachable set and occupancy for all motion primitives



Connectivity check using reachability analysis

Motion Primitives

We use linear models for velocity v and curvature κ:

$$v^*(t) = p_1 + p_3\, t$$
$$\kappa^*(t) = p_2 + p_4\, t.$$

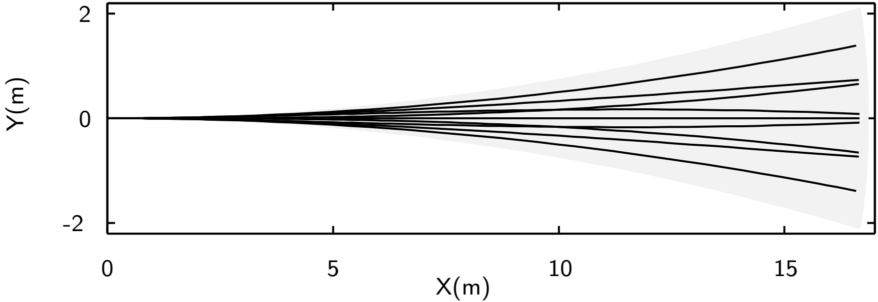
Other states are obtained using a unicycle model:

$$\dot{X}^* = \cos(\theta^*)\, v^*(t)$$
$$\dot{Y}^* = \sin(\theta^*)\, v^*(t)$$
$$\dot{\theta}^* = v^*(t)\, \kappa^*(t)$$

This choice results in spiral trajectories, specifically Euler spirals for the case of $\dot{v}^* = 0$.

Uncertain Motion Primitives

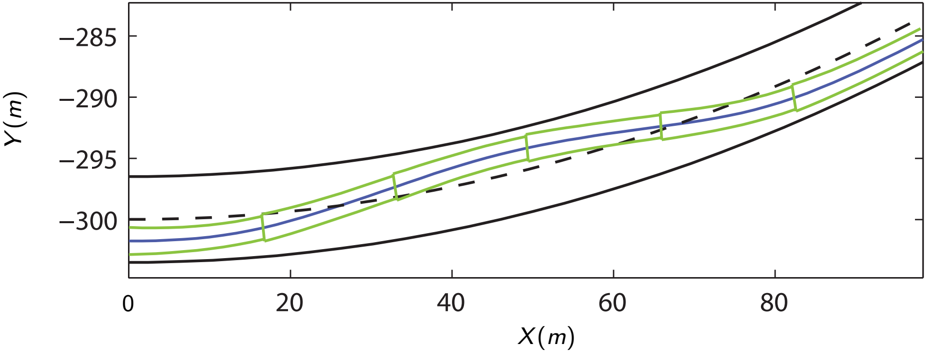
We allow uncertainties for each parameter ($p_i \in [\underline{p}_i, \overline{p}_i]$) so that we can represent infinitely many motion primitives (here: 9 nominal motion primitives):



Results of the pre-computed reachable sets are valid for any motion primitive, where $\forall i: \quad p_i \in [\underline{p}_i, \overline{p}_i]$

Example for Connecting Motion Primitives

- Motion primitives are connected according to the connection constraint.
- The reachability analysis is performed in almost no time due to the precomputation.



Interaction between Planning and Verification

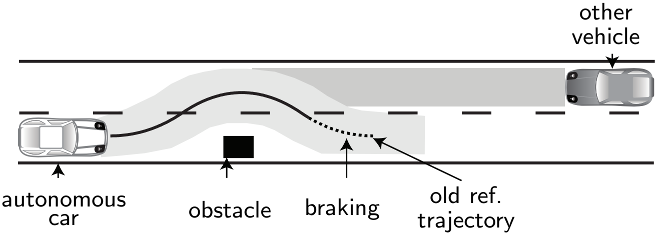
Planning Challenges

- What if the reference trajectory is unsafe?
- Is there enough time to re-plan and verify a new trajectory?
- What if a software bug or hardware failure occurs?

Planning Solution

- Plan maneuvers that are safe for all times (details later).
- Only change the previous plan if the new plan has already been verified.

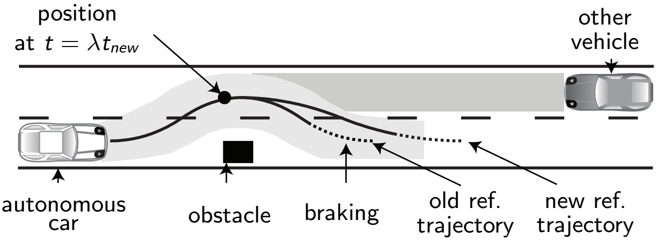
Maneuvers Verified for all Times



General Idea

- Add a braking maneuver to the end of the originally intended maneuver.
- The vehicle has to stop in a safe location (e.g. not on a railway crossing).
- The additional braking trajectory is only executed when no new safe plan is ready for execution.

Deviation from Previous Plan



General Idea

- Change maneuver only at a point from where the new reference trajectory has been verified.
- Verification time is linear in the time horizon t_f: $t_{ver} = \lambda t_f$.

$\rightarrow$ Change previous plan at $\lambda t_{f,\mathrm{new}}$.

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